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14. ABSTRACT
Stochastic semidefinite programs (SSDP's) are a new class of optimization problems with a wide variety applications proposed by the PI and his doctoral students. The broad objective of this project was to develop applications of and algorithms for SSDP's. We have developed five classes of novel applications, and three classes of new algorithms. We have proved the convergence and polynomial complexity of the algorithms. We have also identified two new classes of optimization problems which may be useful for future research.

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					509-335-3152

Report Title

Stochastic Semidefinite Programming: Applications and Algorithms

ABSTRACT

Stochastic semidefinite programs (SSDP's) are a new class of optimization problems with a wide variety applications proposed by the PI and his doctoral students. The broad objective of this project was to develop applications of and algorithms for SSDP's. We have developed five classes of novel applications, and three classes of new algorithms. We have proved the convergence and polynomial complexity of the algorithms. We have also identified two new classes of optimization problems which may be useful for future research.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
2012/03/30 1!	7	K. A. Ariyawansa, Yuntao Zhu. A class of polynomial volumetric center decomposition algorithms for stochastic semidefinite programming, Mathematics of Computation, (07 2011): 1639. doi:
2011/08/30 1!	6	Yuntao Zhu, K. A. Ariyawansa. A preliminary set of applications leading to stochastic semidefinite programs and chance-constrained semidefinite programs, Applied Mathematical Modelling, (11 2010): 2425. doi:

TOTAL: 2

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received		Paper
2011/09/07 1:	13	K. A. Ariyawansa, Yuntao Zhu. A Class of Polynomial Volumetric Barrier Decomposition Algorithms for Stochastic Semidefinite Programming, Mathematics of Computation, (07 2011): 1639. doi:

TOTAL: 1

Number of Papers published in non peer-reviewed journals:

(c) Presentations

Baha M. Alzalg and K. A. Ariyawansa, Stochastic symmetric programming over integers. International Conference on Scientific Computing, Las Vegas, Nevada, July 18--21, 2011.

Baha M. Alzalg. On recent trends in stochastic conic optimization.
INFORMS Annual Meeting, Charlotte North Carolina, November 13--16, 2011.

K. A. Ariyawansa, Stochastic semidefinite programming,
Invited Presentation at Computer and Information Sciences Division,
US Army Research Laboratory, Adelphi, Maryland, February 2, 2012.
This visit to the CISD of ARO provided the opportunity to interact with CISD scientists. In particular, interactions with CISD scientists Ananthram Swami and Brian Sadler were very useful.

Number of Presentations: 3.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received	Paper
2011/08/30 2(9	Baha M. Alzalg, K. A. Ariyawansa. Stochastic mixed integer second-order cone programming: A new modeling tool for stochastic mixed integer optimization, WORLDCOMP'11. 2011/07/18 03:00:00, . : ,

TOTAL: **1**

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received Paper

TOTAL:

Number of Manuscripts:

Books

Received Paper

TOTAL:

Patents Submitted

Patents Awarded

Awards

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
Baha M. Alzalg	0.50	
Limin Yang	0.00	
Bo Han	0.00	
FTE Equivalent:	**0.50**	
Total Number:	**3**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
K. A. Ariyawansa	0.15	
FTE Equivalent:	**0.15**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: 0.00

Names of Personnel receiving masters degrees

NAME
Total Number:

Names of personnel receiving PHDs

NAME	
Baha M Alzalg	
Total Number:	1

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

Please see Attachment.

Technology Transfer

ARO PROPOSAL NO. 50556-MA, ARO AWARD W911NF-08-1-0530
SCIENTIFIC PROGRESS AND ACCOMPLISHMENTS
during the Period September 26, 2008–December 31, 2011

This project was on stochastic semidefinite programming, a class of new stochastic optimization problems proposed by the PI and his former graduate student Yuntao Zhu [2]. The motivation to propose stochastic semidefinite programs (SSDP's) arose as a result of the following line of argument. Semidefinite programs (DSDP's), defined using deterministic data, have been the focus of intense research during the past 15 years. DSDP's generalize (deterministic) linear programs (DLP's). In particular, the decision variable of a DLP is a nonnegative vector, while the decision variable of a DSDP is a positive semidefinite matrix. The generalization of DLP's to DSDP's remarkably increases the applicability of DLP's. Indeed, many optimization problems that have been considered to be intractable prior to the prominence of DSDP's have been formulated and solved as DSDP's. There are efficient algorithms for solving DSDP's, and they are almost exclusively based on interior point concepts.

An equally applicable extension of DLP's are stochastic linear programs (SLP's) with recourse. SLP's are an important (but not the only) way to deal with uncertainty in data defining DLP's, and as in DLP's the decision variable in a SLP is a nonnegative vector. There are efficient interior point and noninterior point algorithms for solving SLP's.

Since DSDP's (with deterministic data and positive semidefinite matrix decision variables) and SLP's (with stochastic data and nonnegative vector decision variables) are both very useful extensions of DLP's, it is desirable to seek an extension that combines them. SSDP's defined in [2] are such an extension (with stochastic data and positive semidefinite decision variables) of DLP's.

The relationships among DLP's, DSDP's, SLP's and SSDP's indicated above are illustrated in Figure 1 in page D-9 of the Proposal 50556-MA.

In very broad terms the objective of this project was to investigate applications of SSDP's, and to derive, analyze and computationally test algorithms for SSDP's. Specific tasks are stated in pages D-18 and D-19 of the Proposal 50556-MA.

Our descriptions here are brief and semitechnical, and use symbols and terms defined in the Proposal 50556-MA. Technical reports that have been submitted to ARO, and also to peer-reviewed journals for consideration for publication, provide precise technical descriptions.

While performing Task 1, it became apparent that a new alternative to SSDP for handling the uncertainty in data defining DSDP's may be formulated. We begin by describing this alternative to SSDP's.

(a) Accomplishment 1

Chance-constrained linear programming (CCLP) [10, 11, 14, 15] is a prominent alternative to SLP for handling uncertainty in data defining DLP's. Briefly, a constraint requiring decision variable $x \in \mathbb{R}^n$ to be chosen such that $a^\mathsf{T} x \le b$ makes sense when data $a \in \mathbb{R}^n$ and $b \in \mathbb{R}$ are deterministic. However, when a and b are random such a constraint loses its meaning, and in CCSLP it is replaced by a *chance-constraint* $P(a^\mathsf{T} x \le b) \ge p$ which requires that x be chosen so that the probability of the event $\{x \in \mathbb{R}^n : a^\mathsf{T} x \le b\}$ is at least a prescribed value $p \in (0, 1)$. We have developed a class of new optimization problems termed chance-constrained semidefinite programs for handling uncertainty in DSDP's. CCSDP's are related to DSDP's in the same way that CCLP's are related to DLP's, and CCSDP is an alternative to SSDP for handling uncertainty in data defining DSDP's. See [3] for details.

In order to demonstrate the applicability of SSDP, in [2], the authors showed how an SSDP may be formulated for dealing with the uncertainty in data defining the minimum-volume covering ellipsoid problem [18, 16]. As indicated in [2], a concrete application of this SSDP model of the random minimum-volume covering ellipsoid problem is in optimally tracking and destroying randomly moving targets. In [3, §4], we show how a CCSDP model can also be formulated to handle the uncertainty in data defining the minimum-volume covering ellipsoid problem. Further, we show how this CCSDP model of the random minimum-volume covering problem leads to a different model for optimally tracking and destroying randomly moving targets.

(b) Accomplishment 2

This is based on Task 1 of the proposal. This task is on identifying applications where SSDP and CCSDP models can be developed to deal with uncertainty not captured by existing approaches. We have identified three such applications and developed corresponding new SSDP and CCSDP models. Specifically, we have developed the following SSDP and CCSDP models:

(i) an SSDP model for determining routing strategies in mobile ad-hoc networks;

(ii) an SSDP model and a CCSDP model for designing RC circuits;

(iii) an SSDP model and a CCSDP model for structural optimization;

(iv) an SSDP model for portfolio optimization under risk constraints. This is joint work with doctoral student Baha Alzalg supported by this grant.

Along with the SSDP and CCSDP models for the random minimum-volume covering ellipsoid problem developed in [2] and [3] respectively, we thus have new models for capturing uncertainty in five application areas. Complete details of the SSDP an CCSDP models in the first four of these five areas are given in [4].

(c) Accomplishment 3

This is based on Task 2. The purpose of this task was to develop and analyze algorithms for the generic SSDP with finite Ω and large K. We have developed such a class of algorithms with the following features.

(i) The class of algorithms is of decomposition type as described in Part (2.2) of Task 2 in page D-18 of Proposal 50556-MA. It decomposes the large scale problem into K smaller independent subproblems. Thus implementations of members of the class can utilize parallel processing naturally.

(ii) It is based on the volumetric barrier function [17, 1]. It is known that interior point algorithms based on the volumetric barrier perform better than comparable algorithms based on the standard logarithmic barrier. It is due to this reason that our derivation of algorithms was based on the volumetric barrier. However, analyses of algorithms based on the volumetric barrier are technically much more challenging than analyses of algorithms based on the logarithmic barrier.

(iii) We have analyzed the class of algorithms and proved convergence and polynomial complexity of certain members of the class.

Complete details of (i), (ii) and (iii) may be found in [5].

(d) Accomplishment 4

This is based on work in Task 2. The purpose of this task was to develop and analyze algorithms for the generic SSDP with finite Ω and large K, the case that is useful in solving problem instances that arise in applications. In this case, the generic SSDP reduces to a large-scale DSDP with very special structure. Therefore, it is possible to solve the DSDP by applying a general-purpose algorithm for DSDP's. However, when the number of realizations K is large, this is prohibitively expensive.

More efficient algorithms can be derived by exploiting the special structure in the DSDP that the generic SSDP reduces to when Ω is finite and K is large. In Task 2 it was proposed (see page D-18 of the Proposal 50556-MA) to develop such efficient algorithms in two different ways: by tailoring general purpose DSDP algorithms to take the special structure into account; and by generalizing decomposition algorithms for SLP's such as the one in [23].

Accomplishment 3 was on developing a class of algorithms of the latter kind (see [5]). In particular, in [5] it was shown that the computational complexity (in terms of arithmetic operations) of the long-step members of the class of algorithms is $O(K^2)$. This is much smaller than the complexity of $O(K^5)$ of corresponding long step algorithms that ignore the special structure of SSDP's with finite Ω and large K.

This accomplishment is on deriving two classes of algorithms of the first kind, and proving their convergence and polynomial complexity. The first class is based on the homogeneous self-dual algorithms [12] for DSDP's and extends the work of Berlelaar, Kouwenberg and Zhang [9] for SLP's to SSDP's. The complexity of the long step members of the class of algorithms is $O(K^2)$ in terms of arithmetic operations. Our class of algorithms has two important features. First, being homogeneous self-dual algorithms, they have a standard starting point, and if the problem is infeasible the algorithms will terminate indicating infeasibility. All interior point algorithms need a strictly feasible starting point, and in practice, feasibility is assumed, and an initialization phase is used to find a suitable starting point. The second feature of our algorithms is the possibility for organizing the computation of the search direction (the most expensive part of the algorithm) into K smaller computations. This feature allows easy distributed processing in implementations.

The second class of algorithms we have developed is based on the infeasible interior point algorithms [13] for DSDP's. The complexity (in terms of arithmetic operations) of the class is $O(K^{1.5})$ if the starting point is feasible or close to being feasible and $O(K^2)$ otherwise. The most expensive computation of this class also naturally decomposes into K smaller computations allowing distributed processing in implementations.

(e) Accomplishment 5

Much of the work in Tasks 1, 2, 3, 4 of the pages D-18, D-19 of the Proposal 50556-MA assumes that Ω is finite leading to the formulation of SSDP's in equation (14) of the proposal. While this is almost always true in applications, the general formulation of the SSDP in equations (11,12,13) of the proposal does not require this assumption. For example, it may be the case that Ω is not finite and the probability distributions of random data is continuous. In such cases one approximates the continuous probability distribution by a discrete distribution with a finite number of realizations. It is then natural to repeat this process with a sequence of distributions with increasing numbers of realizations converging to the continuous probability distribution. This leads to the solution of a sequence of problems of the form in equation (14) of the proposal, and questions on the convergence of the sequence of solutions to a solution to the original problem arise. For purposes such as this, the general problem in equations (11,12,13) of the problem must be studied. These theoretical questions that provide the foundations to Tasks 1, 2, 3, 4 are studied in the doctoral dissertation of Mr. Limin Yang supported by this grant. Mr. Yang will have his final doctoral examination in Summer 2012. This dissertation contains four chapters titled:

(i) Stochastic Semidefinite Programs: The Equivalent Convex Program

(ii) Stochastic Semidefinite Programs: The Solution Set

(iii) Stochastic Semidefinite Programs with Recourse: General Properties

(iv) Stochastic Semidefinite Programs with Recourse: On the Continuity of the Objective.

The results in these four chapters extend the results in [19, 20, 21, 22] for SLP's to the case of SSDP's.

Accomplishment 6.

While working on Task 3 it became apparent to us that all applications of SSDPs that we were aware of belong to a proper subclass of SSDPs that we termed Stochastic Second-Order Cone Programs (SSOCPs). SSOCPs are obtained from SLPs by replacing linear inequalities by second-order cone (SOC) inequalities, and SSDPs are obtained from SOCPs by replacing SOC inequalities by linear matrix (LM) inequalities. Finally, replacing LM inequalities by what we term symmetric cone (SC) inequalities we get stochastic symmetric programs (SSPs). The hierarchy of problems SLPs, SSOCPs, SSDPs and SSPs have deterministic counterparts, DLPs, DSOCPs, DSDPs and DSPs respectively. In applications, it is useful to be able to restrict some variables to take integer values while others take real values as usual. We refer to such problems as mixed integer (MI). The four problems DLPs, DSCOPs, DSDPs and DSPs can have mixed integer counterparts DMILPs, DMISCOCPs, DMISDPs, and DMISPs respectively. These relationships are illustrated in Figure 1. The boxes with dashed boundaries represent *new* practically useful problems requiring significant work on algorithms for their solution.

Accomplishment 7.

The discovery of SSOCPs led to another accomplishment. Second order problems use ℓ_2 norm exclusively. In practical settings it may be necessary to use ℓ_p norms with p values other than 2. It seems much of the of the present proposal could be extended to problems that use ℓ_p norms with any $p \in [1, \infty)$. We refer to such problems as p-th order problems. We then realized that it is possible to construct problems that are over the intersection of a finite number of cones with different p values. We term these problems as *multi-order cone programs* (SMOCPs). As in Accomplishment 1, we can have both deterministic and mixed integer versions. This set of problem classes are illustrated in Figure 2.

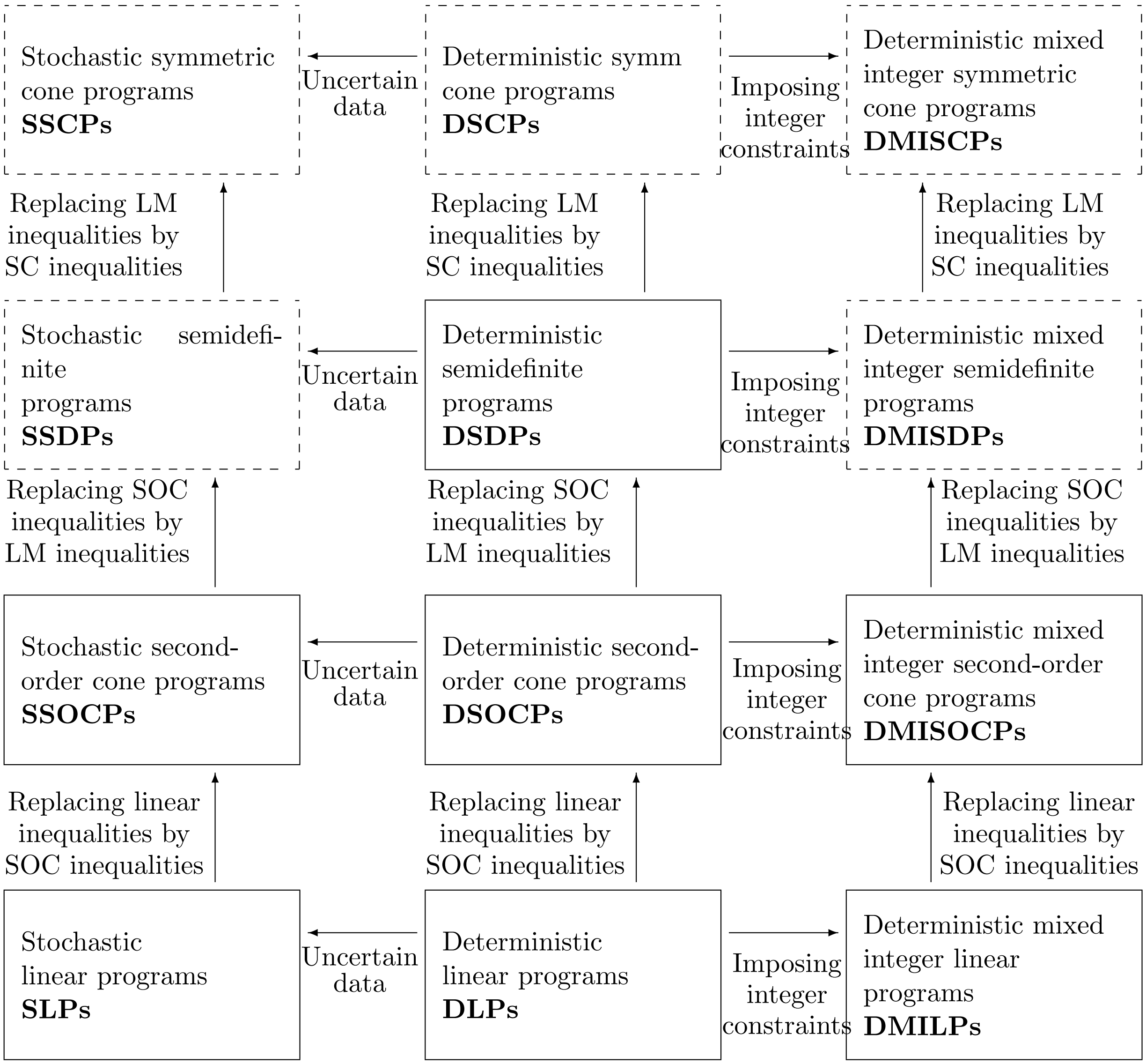

Figure 1: Conceptual relationships among optimization problems over symmetric cones and their special cases.

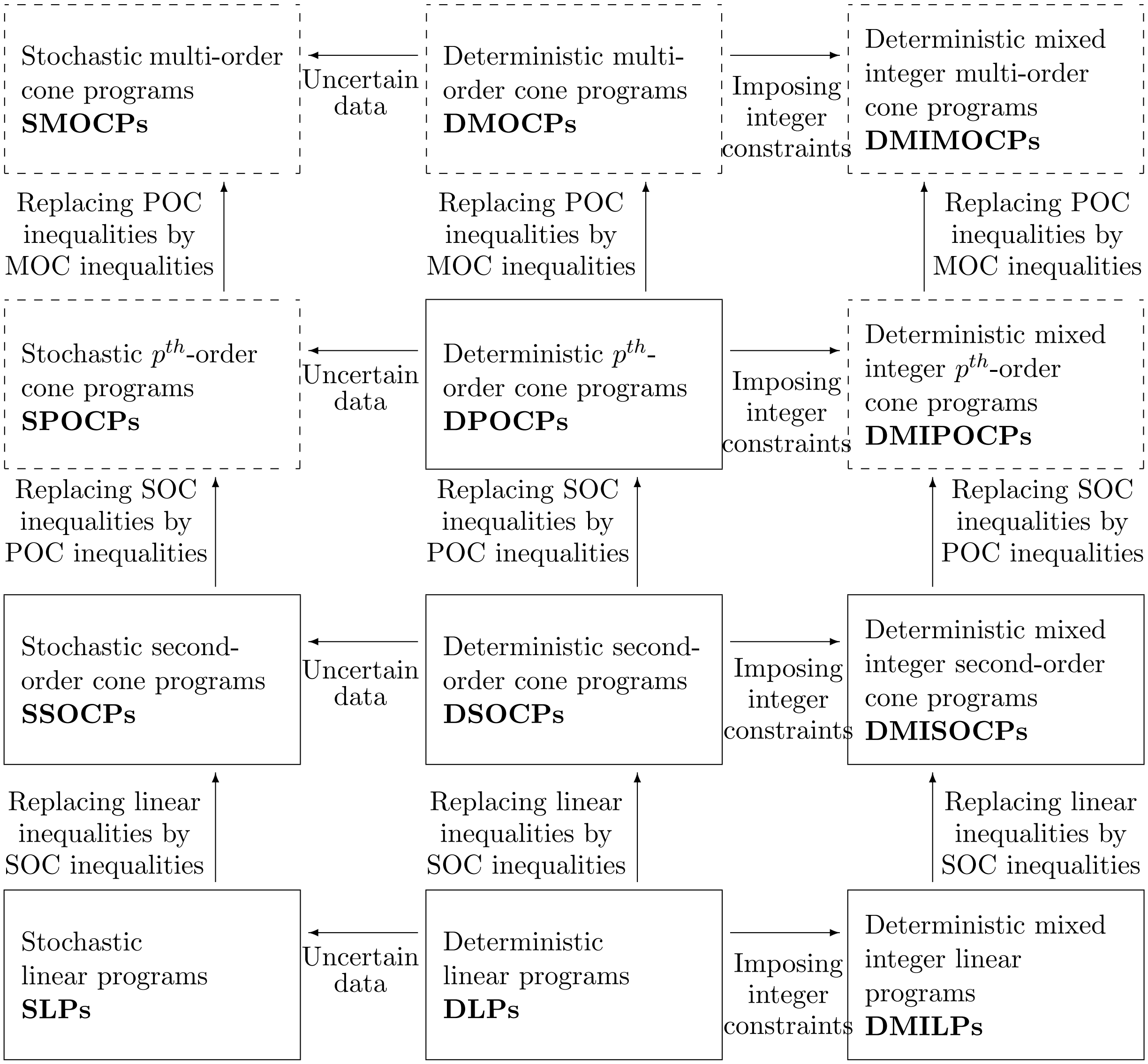

Figure 2: Conceptual relationships among the optimization problems over multi-order cones and their special cases.

As in Figure 1, the boxes with dashed boundaries represent *new* classes of optimization problems. Modeling, algorithmic, and theoretical work based on the problems in boxes with dashed boundaries would be suitable for projects following the present project.

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